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Sampling of variables with proportional effect

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ABSTRACT

According to the theory of regionalized variables, a variogram can be applied for local variations only if the second-order stationarity condition is satisfied; which means that variance and covariogram are stationary and there is no drift.

However in virtually every type of ore deposits, Au presents log-normal distributions and consequently the so-called "proportional effect" (PE); which is considered a clear sign of non-stationarity. This characteristic is not exclusive to gold, because we have recognized PE in other elements such as Ag, Cu, and Mo.

The best way to recognize PE is plotting, in a correlation XY diagram, mean vs standard deviation of different sub-domains of the studied area; if the linear correlation is high and positive, then unquestionably we are facing a variable with PE. We presented several examples of peruvian HS Au epitermal deposits and porphyry Cu-Mo deposits, in which we have found variables with PE.

In domains with this type of variables is not possible to apply traditional geostatistics. We discussed the proposed solutions by different authors to handle PE; and in addition we suggested a very simple one, which consists in dividing domains into appropriate subdomains, to minimize the proportional effect, working within each subdomain applying their more suitable structural analysis and kriging.

We additionally recommend, for high graded subdomains, and consequently of high variance, a higher sampling density, greater support for the sample (larger mass or weight) and greater number of increments; as well as chemical analysis of large support as BLEG.

INTRODUCTION

In exploration geochemistry, elements in the order of traces, usually present log-normal distributions, which are not symmetrical but positively skew. Some elements even exhibit this behaviour at ranges greater than trace and up to ranges of economic values. It is the case of gold that usually has this feature, regardless of the range of their values or type of deposit.

The objective of this paper, is to draw attention to this fact, because log-normal distributions are associated to the so-called "proportional effect" (PE), that is precisely the opposite situation of the stationary condition that traditional geostatistics arises as a requirement for implementing the variogram function.

To illustrate this paper we are including figures 1, 2, 6 and 7, which are examples of the a HS epithermal gold deposit located in Tarata-Tacna; while figures 3, 4 and 5 are examples of other type of elements and deposits.

VARIOGRAM AND THE STATIONARITY CONDITION

In traditional geostatistics, as is pointed out by Matheron (1963), David (1977), Journel & Huijbregts (1991) and others, the variogram can be used for local valuation only if the "second order stationarity condition" is satisfied.

In a domain *D*, we can have data for one type of measurement ***z*** at various locations and construct at each of the ***n*** locations a random variable *Z(x)*; further assume that these random variables are a subset of an infinite collection of random variables called random function *Z(x)* defined at any location *x* of the domain *D*.

We assume the random function is second-order stationarity if the expectation *E* and the covariance *Cov* are both translation invariant over the domain, i.e. for a vector *h* linking any two points *x* and *x+h* in the domain:

$$E[Z(x+h)] = E[Z(x)] \quad (1)$$

$$Cov[Z(x+h), Z(x)] = C(h) \quad (2)$$

In other words, the expected value *E[Z(x)] = m* is the same at the any point *x* of the domain, and the covariance between any pair of locations depends only on the vector *h* which separate them. In geological terms this means that, within a domain *D*, the structure of the variability between two grades *z(x)* and *z(x+h)* is constant and, thus, independent of *x*. This would be true only if the mineralization within *D* were probabilistic homogeneous.

The variability between two numerical values *z(x)* and *z(x+h)* is characterized by the variogram function $2\gamma(x,h)$, which is defined as the expectation of the random variable $[Z(x) - Z(x+h)]^2$:

$$2\gamma(x,h) = E\{[Z(x) - Z(x+h)]^2\} \quad (3)$$

In practice applications we need to introduce the intrinsic hypothesis, which states that the variogram function $2\gamma(x,h)$ depends only on the separation vector *h* and not on the location *x*. It is then possible to estimate the variogram $2\gamma(h)$ from the available data using an estimator $2\gamma^*(h)$ defined us:

$$2\gamma^*(h) = \frac{1}{n(h)}\sum_{i=1}^{n(h)}[z(x_i) - z(x_i + h)]^2 \quad (4)$$

Where *n(h)* is the number of experimental pairs $[z(x_i) - z(x_i + h)]$ of data separated by the vector h.

SCOPE AND DEFINITION OF PROPORTIONAL EFFECT

Log-normal distributions

Experience shows that in most situations, and certainly in the case of low-grade deposits, assay values do not follow a normal distribution, but rather the distribution of the raw data is markedly positive skewed, as shown in Fig. 1. The general equation of the distribution is:

$$f(x) = \frac{1}{x\beta\sqrt{2\pi}} \exp[-\frac{1}{2}\left(\frac{\ln\ \mu - \ln\ x}{\beta}\right)^2] \quad (5)$$

Where the median of the distribution is: $\mu = e^{\alpha}$, if α is the average of the logarithms, and β their standard deviation (David, 1977; Krige, 1981 and others).

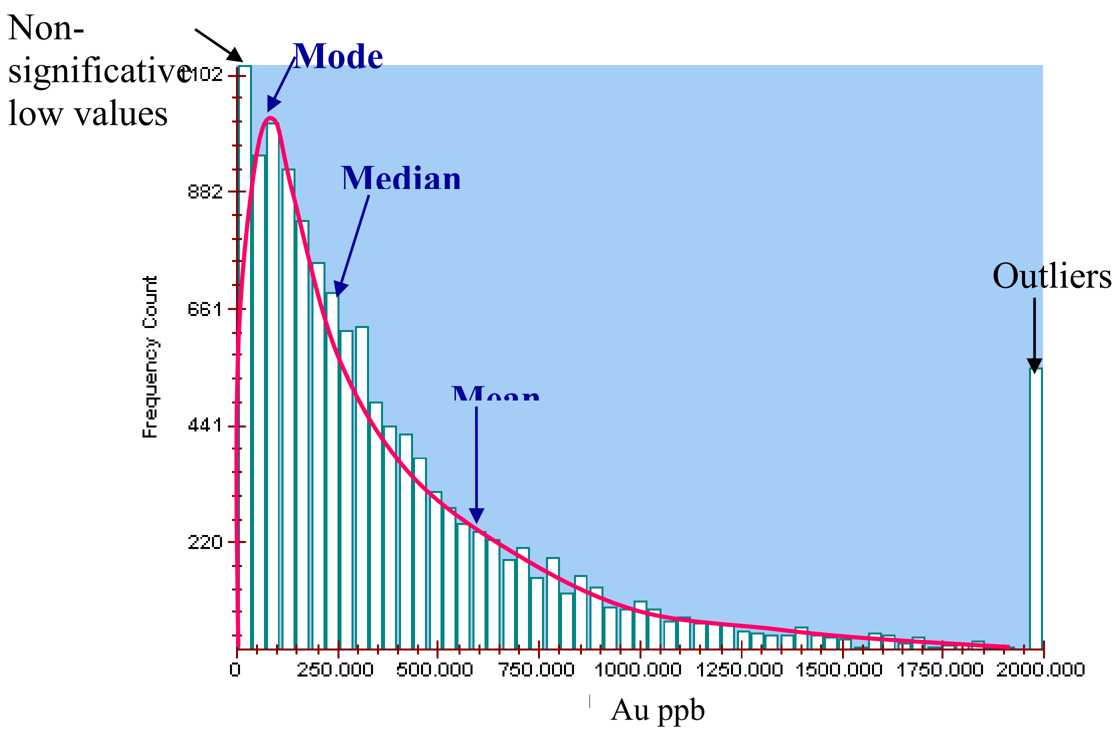

Figure 1 Typical log-normal histogram. HS Epithermal Gold Deposit; Palca, Tacna.

Log-normal distributions are associated to the so-called "proportional effect" (Figs. 2, 3, 4 and 5), that is precisely the opposite situation of the stationarity condition that geostatistics arises as a requirement for implementing the variogram function.

Definition of the Proportional Effect

Consider a domain *D* subdivided into sub-domains D_i, there we have calculated their respective means μ_i and standard deviations σ_i. By plotting μ_i vs σ_i we get a scatter plot that is adjusting well to a straight line (Fig 2), linear equation is included; which is the irrefutable evidence that we are up against the so-called: "proportional effect" (EP). Virtually all gold deposits exhibit this characteristic; although it has also been observed in other elements, and in different types of deposits (Figures 3, 4 and 5).

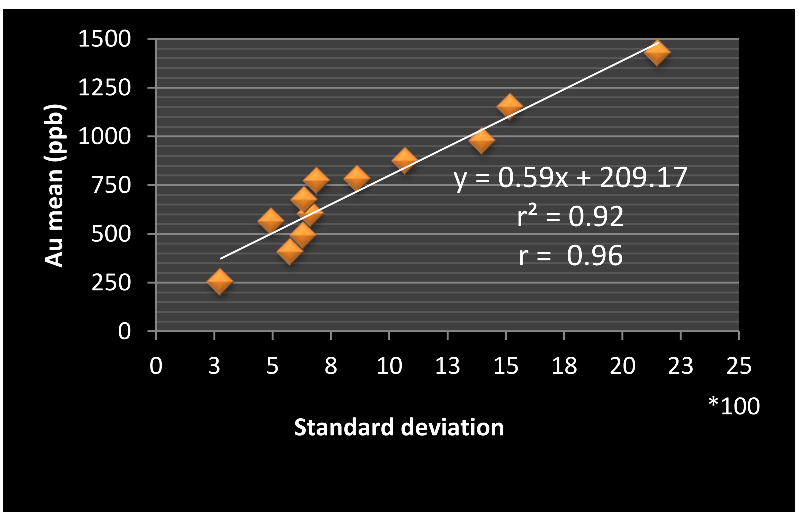

Figure 2 Scatter plot for gold: standard deviation vs mean obtained from selected DDH from Au HS Epithermal Gold Deposit; Palca, Tacna.

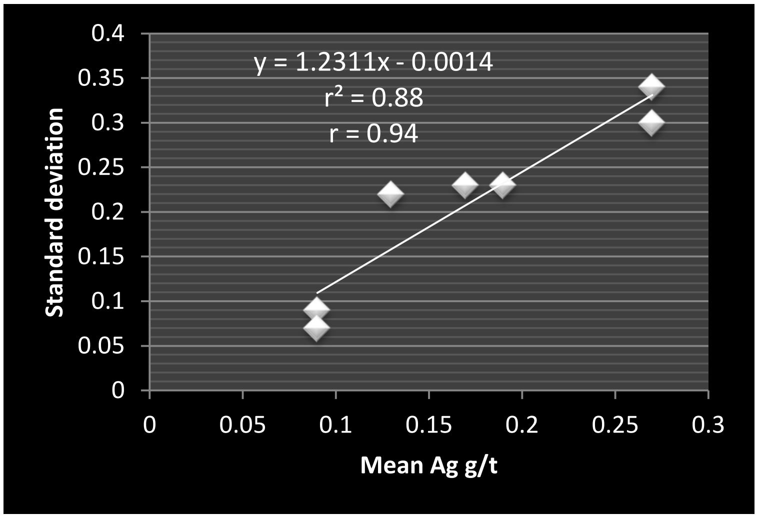

Figure 3 Scatter plot for silver: standard deviation vs mean. Ag-Au Deposit; Maricunga Belt - Chile.

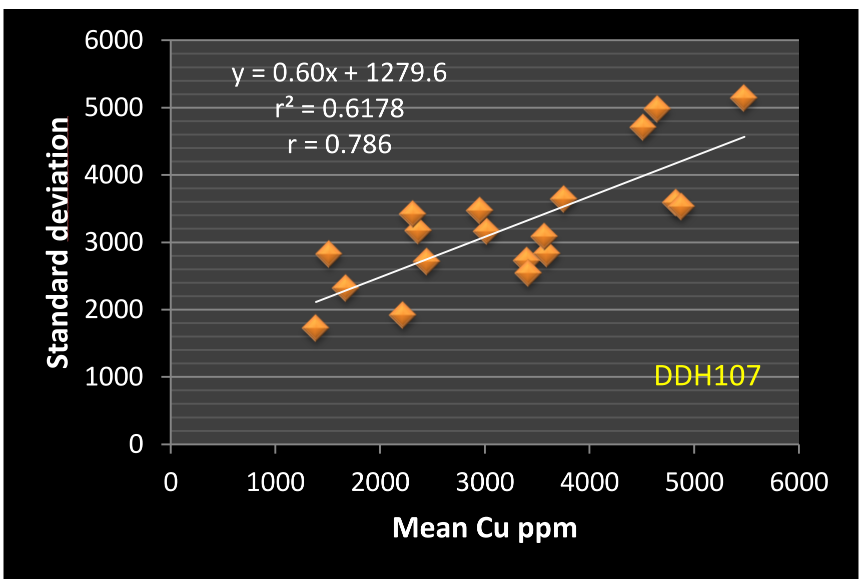

Figure 4 Scatter plot for copper: standard deviation vs mean. Cu-Mo Trapiche Porphyry - Apurímac.

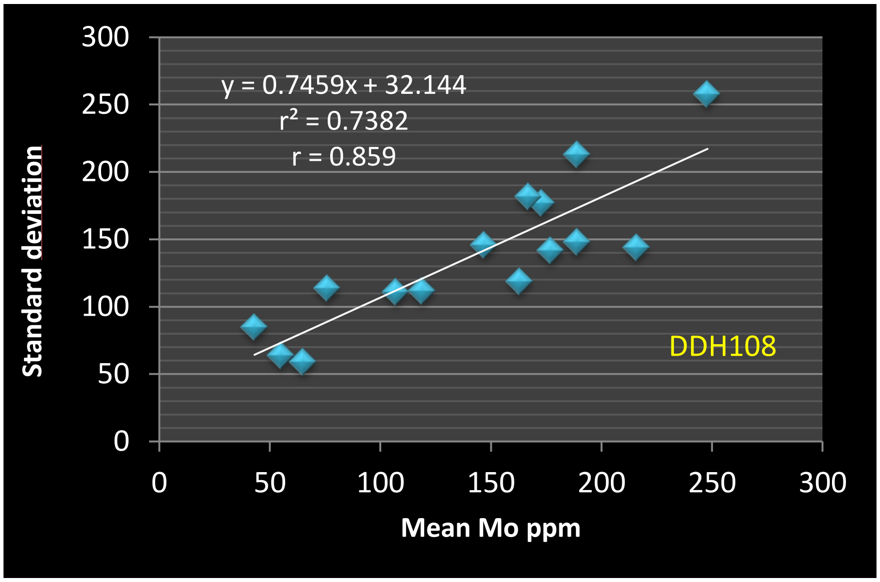

Figure 5 Scatter plot for molybdenum: standard deviation vs mean. Cu-Mo Porphyry, Trapiche - Apurímac.

The main concern related to the variables with EP is that in the domains where this characteristic occur, it is clear that the higher the local mean then less precise estimatations will be.

EXPRESSION OF PROPORTIONAL EFFECT ON VARIOGRAMS

When plotted the experimental variograms of some drills considered in Figure 2, the proportional effect is clearly reflected in them (Fig. 6): the sill of the variograms (i.e. the variance of samples) is proportional to the square of the respective means. In order to show it clearly, we have only traced four variograms.

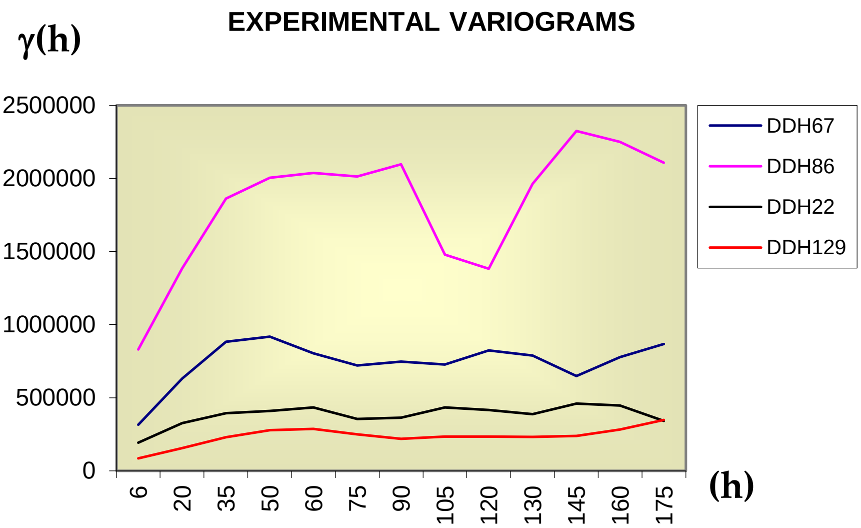

Figure 6 Experimental variograms of four selected holes showing the proportional effect. Au HS Epithermal Gold Deposit; Palca, Tacna.

DISCUSSION

In the bibliography are controversial recommendations to deal with distributions with proportional effect. For example it is advisable to divide the gamma values of each variogram by their respective squared mean; so it gets the called "relative variogram". It should be noted that doing this, the resulting values are dimensionless (do not have units); to convert them in significant digits must be multiplied by the square of the respective mean.

Delhomme (1978) proposed equation 6 to model rainfall with PE where

$$\gamma_k(h) = \frac{s_k^2}{s^2}\bar{\gamma}(h) \qquad (6)$$

After Chiles and Delfiner (2001) equation 7 can be used for modeling variograms of variables with PE, weighting the global variogram (i.e. non conditional stationary random function) by the square of local means. V_o is the neighborhood of point x_o; b is some function of the local mean m_{Vo} and γ(h) a global model. Matheron (1974) shows that if the spatial distribution is lognormal, the local variogram γ_{Vo} takes on the form of equation 7 with $b(m_{Vo})=m_{Vo}^2$

$$\gamma_{Vo}(h) = b(m_{Vo})\gamma(h) \qquad (7)$$

David (1977) and other authors show examples where the relative variograms match, making "disappear" the proportional effect; even recommend replacing all the variograms for one average for being able to model the entire deposit. After Clark (1984) this option is questionable, because when it comes to check locally the values that reproduced the "relative variogram model", they are wrong. Additionally, as pointed out by Canchaya (2004), relative variograms not necessarily get a matching, as show in figure 7, where the relative variograms corresponding to those of the figure 6 has been ploted, getting no "disappearance" of the proportional effect.

After Krige (1981) a logarithmic transformation of the variable will "eliminate" the PE; however suitable adjustments, based on the square of the "local" mean grade, must to be made if the analysis is done on untransformed values (David 1977).

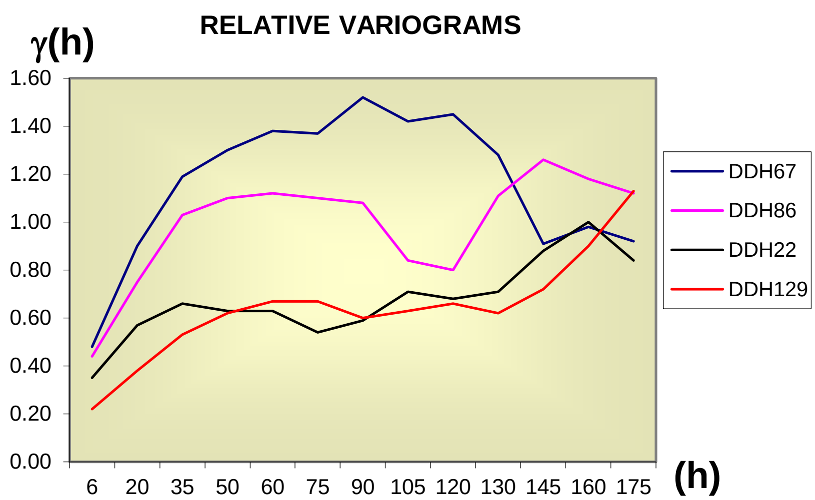

Figure 7 Relative variograms of the same holes of fig. 8, showing no matching.

A practical recommendation to deal with variables with proportional effect (Canchaya 2008) consists in trying to sub-divide the total domain of data in sub-domains of high, middle, and low-grade; and then adjust separated experimental variograms for each sub-domain.

It should be noted that the proportional effect is also reflected in the nugget effect, as can be seen in figure 6; this is comprehensible since the nugget effect is in fact the sill of a small-scale structure.

GUIDELINES FOR SAMPLING VARIABLES WITH PROPORTIONAL EFFECT

Following the recommendation to subdivide the entire D in sub-domains Di for a better handling of the proportional effect, the same procedure could be applied for sampling domains with variables with PE.

Specially in high graded sub-domains, and consequently of high variance, it is recommended a higher sampling density, greater support for the sample (larger mass or weight) and greater number of increments; as well as chemical analysis of large support as BLEG (Bulk leach extractable gold).
The idea of proportional sampling is not new; the recommendation is to applied high density sampling in sub-domains where we have higher grades and consequently lesser precision.

High density pilot sampling, usually executed at the beginning of the exploration projects, to define optimum sampling distance, should be the best opportunity to realize if we have PE or not; carefully inspection of the grades distribution, considering also lithology, alteration and structural variations, should give us the possibility to define appropiated sub-domains in order to handeln succesfully variables with PE. Grade charts down hole or equivalent 3D pictures showed currently by powerfull intrinsec 3D model software, like Leapfrog, give us also the possibility to discover if our variable has dual character, that means partly structured and partly stochastic; the first one being usually a trend, while the second one is due to a random fluctuations interpreted us residual components. In this way we open the possibility to applied ancient procedures like universal kriging or its modern vertions, like that suggested by McLennan & Deutch (2008) for oil industry.

CONCLUSIONS

Proportional effect is quite a complicated problem, with controversial solutions, being necessary more investigation. That is the reason why this thematic was selected as one of the most important research lines of the recently constituted CAIG (Centro Académico y de Investigación Geomatemática) at the Universidad Nacional de Ingeniería in Lima.

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